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WAT RATIONAL CUBIC TRIGONOMETRIC BEZIER CURVES AND ITS APPLICATIONS

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ABSTRACT

In this paper, a kind of Rational cubic Bézier curves by combining algebraic polynomials and trigonometric polynomials, using the weight method is developed, named weight algebraic trigonometric (WAT) Rational Bézier curves. Here, weight coefficients are referred as shape parameters, which are called weight parameters. The value of weight parameters can be extended to the interval [0, 1] to [-2, 2.33], and the corresponding WAT Rational Bézier curves and surfaces are defined. The WAT Rational Bézier curves inherit most of properties similar to those of Cubic Bézier curves, and can be adjusted easily by using the shape parameter λ. The jointing conditions of two pieces of curves with the G2 and C^2 continuity are discussed. With the shape parameter chosen properly, the defined curves can express exactly in the form of plane curves or space curves defined by a parametric equation based on {1, Sint, cost, sint2t, cos2t} and circular helix and ellipse, with a high degree of accuracy without using rational form. Examples are given to illustrate that the curves and surfaces, which can be used as an efficient new model for geometric design in the fields of CAGD. It is clear that the existing techniques based on C-Bézier spline can approximate the Bézier curves only from a single side, the WAT Rational Bézier curves can approximate the Bézier curve from both the sides, and the change range of shape of the curves is wider than that of C-Bézier curves. It is important that this WAT Rational Bézier curves much be closer to the control point, compared to the other spline curves. The geometric effect in case of shape preservation of this weight parameter is also discussed.

KEYWORDS: Bézier Curves and Surfaces, Trigonometric Polynomial, Shape Parameter, G^2 and C^2 Continuity

1. INTRODUCTION

Curves and surface design is an important topic of computer aided geometric design (CAGD) and computer graphics. The significance of trigonometric spline in various areas are like in the design of tools and machinery and in the drafting and design of all types of buildings, from small residential types (houses) to the largest commercial and industrial structures (hospitals and factories). Computer aided geometric design (CAGD) studies the construction and manipulation of curves and surfaces using polynomial, rational, piecewise polynomial or piecewise rational splines. Among many generalizations of polynomial splines, the trigonometric splines are of practical importance. In recent years, trigonometric splines with shape parameters have proposed for geometric modeling. A Bézier form of parametric curve is generally used in CAD and CAGD applications like computer animation computational geometry, industrial arts, architectural design, data fitting and font designing. Smooth curve representation of scientific data is also of great interest. In the field of data visualization, it is important that the graphical representation of information is clear and effective. When data arise from a physical phenomenon or problems of industrial design and manufacturing, it is required that the interpolating curve generates a smooth function and has the property like positivity, monotonicity, and convexity. In recent year, various authors have worked in the area of shape preserving splines, using trigonometric rational splines [1-6]. In the recent past, a number of authors and references have contributed to the shape-preserving interpolation and different polynomial methods, which are used to generate the shape-preserving interpolant, have been considered. In this paper, we present a class of new different trigonometric polynomial basis functions with a parameter based on the space Ω=span {1, Sint, cos*t*, sint2*t*, cos2*t*}, and the corresponding curves and tensor product surfaces named WAT Rational trigonometric Bézier curves, and surfaces are constructed based on the introduced basis functions. The WAT Rational Cubic trigonometric Bézier curves not only inherit most of the similar properties to Cubic Bézier curves, but also can express any plane curves or space curves defined by a parametric equation based on {1, Sint, cos*t*, sint2*t*, cos2*t*}, including some quadratic curves such as the circular arcs, parabolas, cardioid exactly and circular helix, with a high degree of accuracy under the appropriate conditions

In this paper, we present a WAT Rational Cubic Trigonometric Bézier curve with weight parameter. The rest of this paper is organized as follows. Section 2, defines the WAT-Bezier Base Functions and the corresponding rational curves and surfaces, also discuss their properties. In section 3, we discussed the shape control of the WAT Rational Cubic Trigonometric Bézier curves. In section 4, Approximation of WAT Rational Cubic Trigonometric Bézier curves to the ordinary WAT Cubic Trigonometric Bézier curves and ordinary Cubic Bézier curves are presented. In section 5, we discuss continuity conditions of WAT-Rational Bezier curves. In section 6, the representations of some curves are shown in figure. Besides, some examples of shape modeling by using the WAT-rational Bezier surfaces are presented. The conclusions are given in section 7.

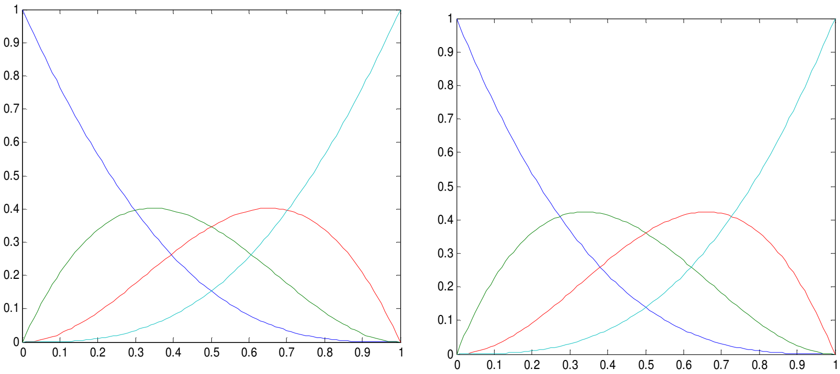
2. WAT-BEZIER BASE FUNCTIONS, WAT-BEZIER CURVES AND SURFACES

2.1 The Construction of the Wat-Bézier Base Functions

Definition 2.1.1: For **$0 \leq \lambda \leq 1$**, and $t \in [0, 1]$, we defined a new basis function are defined as functions

$$
\begin{aligned}
b_0(t,\lambda) &= \lambda(1-t)^3 + (1-\lambda)\frac{\pi(1-t)-\sin\pi t}{\pi}, \\
b_1(t,\lambda) &= 3\lambda(1-t)^2 t + (1-\lambda)\left(\frac{1}{2}+t+\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right), \\
b_2(t,\lambda) &= 3\lambda(1-t)t^2 + (1-\lambda)\left(\frac{1}{2}-t-\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right), \\
b_4(t,\lambda) &= \lambda t^3 + (1-\lambda)\frac{\pi t-\sin\pi t}{\pi},
\end{aligned}
\qquad \textbf{(1)}
$$

These four functions are defined as WAT- Bézier basis. WAT-Bézier basis functions are cubic Bernstein bases when λ =1. And, when λ = 0, WAT-Bézier basis functions are C-Bézier bases.



(a) **(b)**

Figure 1: (a) For Λ = 0.5, (b) For Λ = 0.75

2.1.2 The Properties of the Basis Functions

Theorem 1

The WAT-Bézier base functions (2.1) have the following properties:

1) Properties at the Endpoints

$$b_0(0,\lambda)=1, b_i^{(j)}(0,\lambda)=0,$$
$$b_3(1,\lambda)=1, b_{i-3}^{(j)}(1,\lambda)=0,$$

Where j = 0, 1, 2…i-1, i = 1, 2, 3 and

$$b_i^{(0)}(t,\lambda)=b_i(t,\lambda)$$

2) Symmetry

$$b_1(t,\lambda)=b_2(1-t,\lambda)$$
$$b_0(t,\lambda)=b_3(1-t,\lambda)$$

3) Partition of Unity

$$\sum_{i=0}^{3} b_i(t,\lambda)=1$$

4) Nonnegativity

$$b_i(t,\lambda)\geq 0; i=0,1,2,3.$$

The value of a weight parameter can be extended from interval [0, 1] to $\left[-2,\frac{14\pi^2+8\pi-96}{8\pi-54}\right]$ where $\frac{14\pi^2+8\pi-96}{8\pi-54}=2.33.$

2.2 WAT- RATIONAL BÉZIER CURVES

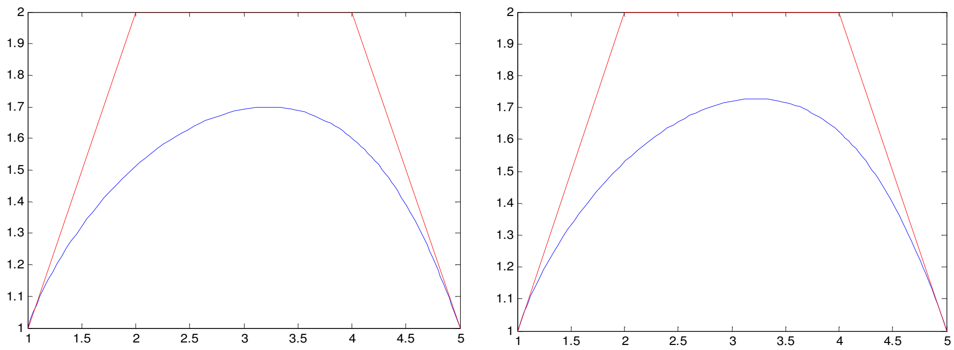
2.2.1 The Construction of the WAT- Rational Bézier Curves

Definition 2.2.1

Given points P_k(k = 0, 1, 2, 3) in R^2 or R^3, then

$$R(t,\lambda)=\frac{\sum_{i=0}^{3}P_i w_i b_i(t,\lambda)}{\sum_{i=0}^{3}w_i b_i(t,\lambda)}\ ;\ t\in[0,1]\quad \text{for}\ \ i=0,1,2,3.\ \ \lambda\in[-2,2.33], \tag{2}$$

This $R(\lambda,t)$ is called WAT- Rational Bezier curve and $b_i(t,\lambda)\geq 0; i=0,1,2,3.$ are the WAT-Bezier basis.



(a) **(b)**

Figure 2: (a) For λ = 0.5, (b) For λ = 0.75

2.2.2 The Properties of the WAT- Rational Bezier Curve

From the definition of the basis function, some properties of the WAT- Rational Bezier curve can be obtained as follows:

Theorem 2

The WAT- Rational Bezier curve curves (2.2.1) have the following properties:

- **Terminal Properties**

$$R(0,\lambda)=P_{0},\ \ R(1,\lambda)=P_{4},$$

$$R'(0,\lambda)=\frac{(\lambda+2)w_1}{w_0}(P_1-P_0)$$

$$R'(1,\lambda)=\frac{(\lambda+2)w_2}{w_3}(P_3-P_2) \tag{3}$$

- **Symmetry:** Assume we keep the location of control points $\boldsymbol{P}_i$ (i = 0,1,2,3,4) fixed, invert their orders, and then the obtained curve coincides with the former one with opposite directions. In fact, from the symmetry of WAT-Bezier base functions, we have

R (1- t, λ, P_3, P_2, P_1, P_0) = R (t, λ, P_0, P_1, P_2, P_3); t ∈ [0, 1], λ∈ [-2, 2.33],

- **Geometric Invariance:** The shape of a WAT- Rational Bezier curve is independent of the choice of coordinates, i.e. (2.2.1) satisfies the following two equations:

  R (t; λ; P_0 + q, P_1 +q, P_2 +q, P_3 +q) = R (1- t; λ; P_3, P_2, P_1, P_0) + q ;

  R (t; λ; P_0 *T, P_1 *T, P_2*T,P_3 *T) = R (1- t; λ; P_3, P_2, P_1, P_0) * T ;

  Where q is an arbitrary vector in R^2 or R^3 and T is an arbitrary d * d matrix, d = 2 or 3.

- **Convex Hull Property:** The entire WAT- Rational Bezier curve segment lies inside its control polygon spanned by P_0, P_1, P_2, P_3.

2.3 WAT Rational Bézier Surfaces

Definition 2.3

Given the control mesh [P_{rs}] (r = i **…**,i + 2; s = j…, j + 2),(i = 0, 1,…, n – 1; j = 0,1,…, m −1), Tensor product WAT- Rational Bézier surfaces can be defined as

$$R_{i,j}(u,v)=\frac{\sum_{r=i}^{3}\sum_{s=j}^{3} w_i b_i(u,\lambda_1) b_i(v,\lambda_2) P_{rs}(u,v)}{\sum_{r=i}^{3}\sum_{s=j}^{3} w_i b_i(u,\lambda_1) b_i(v,\lambda_2)};(u,v)\in[0,1]\times[0,1] \tag{4}$$

Where, $b_i(u,\lambda_1)$ and a $b_i(v,\lambda_2)$ are WAT-Bezier based functions.

3. SHAPE CONTROL OF WAT- RATIONAL BEZIER CURVE

Due to the value of a weight parameter that can be extended from the interval [0, 1] to $\left[-2,\frac{14\pi^2+8\pi-96}{8\pi-54}\right]$, the change range of the WAT- Rational Bézier curve is wider than that of C-Bézier. From the Figure 3, it can be seen that when the control polygon is fixed, by adjusting the weight parameter from -2 to 2.33, the WAT- Rational Bézier curves can cross the cubic Bézier curves and tends to both sides of cubic Bézier curves, the WAT- Rational Bézier curves can range from under the C-Bézier curve to above the cubic Bézier curve. The weight parameters have the property of the geometry. The larger the shape parameter is, and the more approach the curves to the control polygon is. Also, these WAT- Rational Bézier curves we defined include C-Bézier curve ($\alpha = \pi$), as special cases. So that, WAT- Rational Bezier Curve has more advantages in shape adjusting, than that C-Bezier curves do. Also, change the range of the curves is wider than that of C-Bézier curves. The paths of the given curves are line segments. Some transcendental curves can be represented by the WAT with the shape parameters and control points chosen properly. Figure 3 Shows the effect on the shape of WAT-Rational Bezier Curve with altering the values of λ = -2 in green, λ = 0 in red, λ = 1 in blue and λ = 2.33 in yellow.

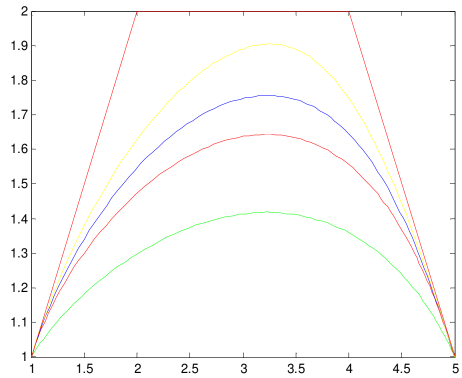


Figure 3: The Effect on the Shape of WAT-Rational Bezier Curve with Altering the Values of λ = -2 in Green, 0 in Red, 1 in Blue and 2.33 in Yellow

4. APPROXIMABILITY

In geometric modeling, Control polygon Play an important tool. Now, we show the relations of the cubic trigonometric rational B-spline curve and cubic Bézier curves corresponding to their control polygon.

Theorem 3

Suppose P_0, P_1, P_2 and P_3 are not collinear; the relationship between cubic trigonometric rational B-spline curve $R(t,\lambda)$ (2.2.1) and the Rational cubic Bézier curve, $t \in [0, 1]$ with the same control points P_i (i=0,1,2,3) are given by

$$B(t) = \frac{\sum_{i=0}^{3} P_i w_i B_i(t)}{\sum_{i=0}^{3} w_i B_i(t)} \tag{5}$$

Where $B_i(t) = \sum_{j=0}^{3} P_i(3,j)(1-t)^{3-j} t^{j}$; are Bernstein polynomial, and the scalar w_i are the weight function

$$R\left(\frac{1}{2},\lambda\right) - P^{*} = a\left[B\left(\frac{1}{2},\lambda\right)\right] - P^{*}; \tag{6}$$

Where $a = \frac{112-70\lambda}{14\pi^2+8(1-\lambda)(\pi+2)}$, $b = \frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}$, and $P^{*} = \frac{P_0+P_3}{2}$

Proof

We assume that $w_0 = w_3 = 1$ and $w_1 = w_2 = 2$ then the ordinary rational cubic Bezier curve (5) takes the form

$$B(t) = \frac{(1-t)^3 P_0 + 6(1-t)^2 t P_1 + 6(1-t)t^2 P_2 + t^3 P_3}{(1-t)^3 + 6(1-t)^2 t + 6(1-t)t^2 + t^3}$$

By simple computation, we have

$$B(0) = R\ (0,\lambda) = P_0;$$
$$B(1) = R\ (1,\lambda) = P_3\ ;$$

$$B(\frac{1}{2}) = \frac{1}{14}(P_0 + 6P_1 + 6P_2 + P_3)$$
$$B(\frac{1}{2}) - P^* = \frac{1}{14}(P_0 - P_1 - P_2 + P_3)$$

Where $P^* = \frac{P_0 + P_3}{2}$

And for $w_0 = w_3 = 1$ and $w_1 = w_2 = 2$, we have

$$R(\frac{1}{2},\lambda)\ -\ P^* = \left[\frac{112-70\lambda}{14\pi^2 + 8(1-\lambda)(\pi+2)}\right] B(\frac{1}{2}) - \left[\frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}\right] P^*$$

$$R(\frac{1}{2},\lambda)\ -\ P^* = a\left[B(\frac{1}{2},\lambda)\right] - P^*;$$

Where $a = \frac{112-70\lambda}{14\pi^2+8(1-\lambda)(\pi+2)}$, $b = \frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}$, and $P^* = \frac{P_0+P_3}{2}$

Equation (6) holds. These equations show that Cubic WAT- Rational Bezier Curve can be made closer to the control polygon by altering the values of shape parameters.

Corollary 3.1: The quadratic, trigonometric Bézier Curves with tension parameter is closer to the control polygon

That the cubic Bezier curve if and only if $\lambda \in \left[-2, \frac{14\pi^2+8\pi-96}{8\pi-54}\right]$

Corollary 3.2: when a = b **= 1** the Cubic WAT- Rational Bezier Curve can be closer to the cubic Rational Bézier Curve, i.e. $B(\frac{1}{2}) = R\ (\frac{1}{2},\lambda);$

Figure 4 shows the relationship among the Cubic WAT- Rational Bezier Curve (yellow line), the cubic Rational Bézier Curve with shape parameter (blue line) and the cubic Bézier Curve (green line).

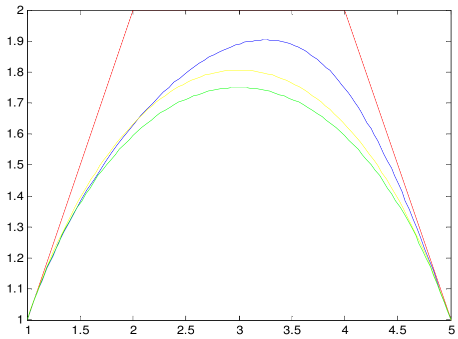


Figure 4: Relationship among the CUBIC WAT- Rational Bezier Curve, the Cubic WAT- Bézier Curve with Shape Parameter and the Cubic Bézier Curve

5. JOINTING OF WAT- RATIONAL BÉZIER CURVES

Suppose there are two segments of WAT- Rational Cubic Bezier curves

$R(t,\lambda_1)=\dfrac{\sum_{i=0}^{3} P_i w_i b_i(t,\lambda_1)}{\sum_{i=0}^{3} w_i b_i(t,\lambda_1)}$ and $Q(t,\lambda_2)=\dfrac{\sum_{i=0}^{3} Q_i w_i b_i(t,\lambda_2)}{\sum_{i=0}^{3} w_i b_i(t,\lambda_2)}$; where $P_3 = Q_0$, parameters of

$R(t,\lambda_1)$ and $Q(t,\lambda_2)$ are λ_1 and λ_2 respectively.

To achieve G^1 continuity of the two curve segments, it is required that not only the last control point of $R(t,\lambda_1)$ and the first control point of $Q(t,\lambda_2)$ must be the same, but also the direction of the first order derivative at jointing point should be the same, namely

$$R^{'}(\lambda,1)=kQ^{'}(\lambda,0) \quad ;(k \geq 1)$$

Substituting Eq. (3) into the above equitation, one can get

$$\frac{w_2}{w_3}(2+\lambda_1)(P_3-P_2) = k\frac{w_1}{w_0}(2+\lambda_2)(Q_1-Q_0)$$

Let $\delta = k\dfrac{w_1 w_3}{w_0 w_2}\left(\dfrac{2+\lambda_2}{2+\lambda_1}\right)$,

Substituting it into the above equitation, then

$$(P_3-P_2) = \delta(Q_1-Q_0)(\delta > 0)$$

Especially, for *k*=1, namely, $\delta = \dfrac{w_1 w_3}{w_0 w_2}\left(\dfrac{2+\lambda_2}{2+\lambda_1}\right)$,

The first order derivative of two segments of curves is equal. Thus, G^1 continuity has transformed into C^1 continuity. Then we can get following theorem 4.

Theorem 4

If $P_{2,}$ P_3 and $Q_{O,}$ Q_1 is collinear and have the same directions, i.e.

$$(\mathbf{P_3} - \mathbf{P_2}) = \boldsymbol{\delta}(\mathbf{Q_1} - \mathbf{Q_0})(\boldsymbol{\delta} > \mathbf{0}) \tag{7}$$

Then curves of $\boldsymbol{P}(t)$ and $\boldsymbol{Q}(t)$ will reach G^1 continuity at a jointing point

And when

$\delta = \dfrac{w_1 w_3}{w_0 w_2}\left(\dfrac{2+\lambda_2}{2+\lambda_1}\right)$ They will get C^1 continuity.

Then we will discuss continuity conditions of G^2 when $\lambda_1 = \lambda_2 = 1$.

First, we'll discuss the conditions of G^2 continuity, which is required to have common curvature, namely

$$\frac{\left|R^{'}(1)\times R^{''}(1)\right|}{\left|R^{''}(1)\right|^{3}}=\frac{\left|Q^{'}(0)\times Q^{''}(0)\right|}{\left|Q^{''}(0)\right|^{3}} \tag{8}$$

Let $\lambda_1 = \lambda_2 = 1$, second derivatives of two segments of curves can get

$$R^{''}(1,\lambda_1)=\frac{1}{w_3^2}\left\{\left[6\lambda_1+\frac{\pi^2}{2}(1-\lambda_1)\right]w_1w_3(P_1-P_3)+\left[\left(-12\lambda_1-\frac{\pi^2}{2}(1-\lambda_1)-2(2+\lambda_1)^2\right)w_2w_3+2(2+\lambda_1)^2w_2^2\right](P_2-P_3)\right\} \tag{9}$$

$$Q^{''}(0,\lambda_2)=\frac{1}{w_0^2}\left\{\left[6\lambda_2+\frac{\pi^2}{2}(1-\lambda_2)\right]w_0w_2(P_2-P_0)+\left[\left(-12\lambda_2-\frac{\pi^2}{2}(1-\lambda_2)-2(2+\lambda_2)^2\right)w_0w_1+2(2+\lambda_1)^2w_1^2\right](P_1-P_0)\right\}$$

Substituting Eq. (3) and (9) into Eq. (8), simplifying it, then

$$\frac{\left|(P_3-P_2)\times(P_2-P_1)\right|}{\left|P_3-P_2\right|^{3}}=\frac{\left|(Q_1-Q_0)\times(Q_2-Q_1)\right|}{\left|Q_1-Q_0\right|^{3}} \tag{10}$$

Substituting Eq. (7) into the above equitation, one can get

$\boldsymbol{h_1 = \delta^2 h_2}$

Where h_1 is the distance from $\boldsymbol{P_1}$ to $\boldsymbol{P_2}$, $\boldsymbol{P_3}$ and h_2 is the distance from $\boldsymbol{Q_2}$ to $\boldsymbol{Q_0}$ $\boldsymbol{Q_1}$. Hence we can get theorem 5.

Theorem 5

Let parameters λ_1**,** λ_2 are all equal one, if they satisfy Eq. (7) and (10), five points $\mathbf{P_1}$**,** $\mathbf{P_2}$**,** $\mathbf{P_3}$**, Q1, Q2** are coplanar and $\mathbf{P_1}$, $\mathbf{Q_2}$ is on the same side of the common tangent, then jointing of curves $R(t,\lambda_1)$ and $Q(t,\lambda_2)$ reach G^2 continuity.

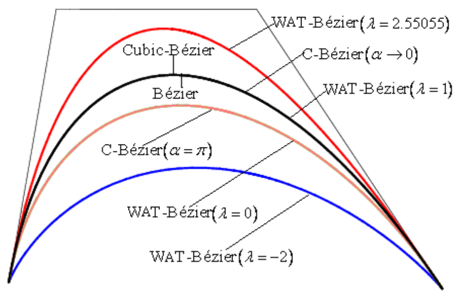


Figure 5: WAT Rational Bezier Curves with Different Values of Shape Parameter

These curves are generated by setting **λ** =-2 in (a), **λ** =0 in (b), **λ** =-1 in (c) and **λ** =2.33 in (d).

6. APPLICATIONS OF WAT- RATIONAL BÉZIER CURVES AND SURFACES

Proposition 4.1

Let $\mathbf{P_0}$**,** $\mathbf{P_1}$**,** $\mathbf{P_2}$ and $\mathbf{P_3}$ be four control points. By taking suitable selection, their coordinates can be expressed in the form

$$P_0=(0,0), P_1=\left(\frac{1-\pi}{2}a, o\right), P_2=\left(\frac{1-\pi}{2}a, 2a\right), P_3=(a,2a)\,(a\neq 0)$$

Then, the corresponding WAT- Rational Bézier curve with the weight parameters $\lambda = 0$ and $t \in [0, 1]$ represents an arc of the cycloid.

Proof

If we take $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ into (2), then the coordinates of the WAT- Rational Bézier curve are

$$x(t) = a(t - \sin \pi t),$$
$$y(t) = a(1 - \cos \pi t)$$

It is a cycloid in parametric form, see Figure 6.

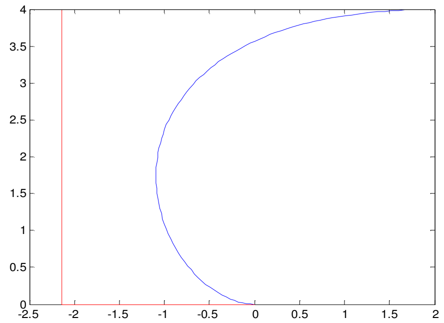


Figure 6: The Representation of Cycloid with WAT- RATIONAL Bézier Curve

Proposition 4.2

Let $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ be four properly chosen control points such that

$$P_0 = (a, 0, 0), P_1 = \left(0, a, \frac{\pi}{2} b\right), P_2 = \left(-a, a, \frac{\pi}{2} b\right), P_3 = (-a, 0, b)(a \neq 0, b \neq 0)$$

Then, the corresponding WAT-Rational Bézier curve with the weight parameters $\lambda=0$ and $t \in [0, 1]$ represents an arc of a helix.

Proof

Substituting $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ into (2) yields the coordinates of the WAT- Rational Bézier curve

$x(t) = a \cos \pi t$,

$y(t) = a \sin \pi t$,

$z(t) = bt$,

Which is the parameters equation of a helix, see Figure 7.

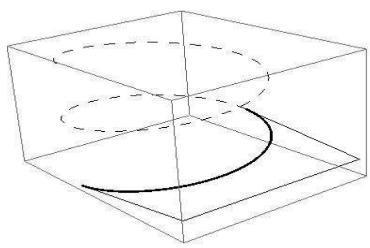


Figure 7: The Representation of Helix with WAT- Rational Bézier Curve

Proposition 4.3

Given the following four control points, $\mathbf{P}_0 = (0,0), \mathbf{P}_1 = \mathbf{P}_2 = (a, \frac{\pi}{2}b), \mathbf{P}_3 = (2a, 0)$ $(ab \neq 0)$, the corresponding WAT-Rational Bézier curve with the weight parameters $\lambda = 0$ and $t \in [0, 1]$ represents a segment of a sine curve.

Proof: Substituting $\mathbf{P_0}$, $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ into (2), we get the coordinates of the WAT- Rational Bézier curve, x (t) = at y (t) = b sin πt, which implies that the corresponding WAT- Rational Bézier curve represents a segment of sine curve, see Figure5.

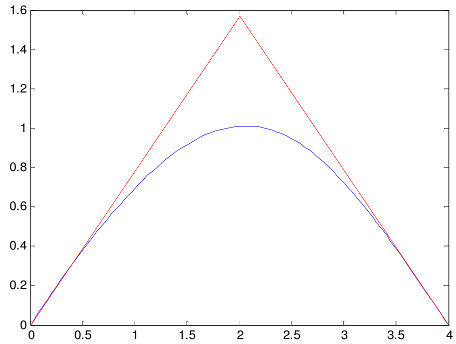


Figure 8: The Representation of Sine Curve with WAT- Rational Bézier Curves

7. CONCLUSIONS

In this paper, the WAT-Rational Bézier curves have the similar properties that cubic Bézier curves have. The jointing of two pieces of curves can reach G^2 and C^2 continuity under the appropriate conditions. The given curves can represent some special transcendental curves. What is more, the paths of the curves are linear, the WAT- Rational Bézier curves have better than C-Bézier curves, in case of shape adjusting. Both rational methods (NURBS or Rational Bézier curves) and WAT- Rational Bézier curves can deal with both free form curves and the most important analytical shapes for engineering. However, WAT- Rational Bézier curves are simpler in structure and more stable in the calculation. The weight parameters of WAT- Rational Bézier curves have geometric meaning and are easier to determine than the rational weights in rational methods. Furthermore, some complex surfaces can be constructed by these basic surfaces, exactly. The method of traditional Cubic Bézier curves needs joining with many patches of surface in order to satisfy the precision of users for designing. Therefore, the method presented by this paper can raise the efficiency of constituting surfaces and precision of representation, in a large extent. Meanwhile, WAT- Rational Bézier curves can represent the helix and the cycloid precisely, but NURBS can not. Therefore, WAT- Rational Bézier curves would be useful for engineering.

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